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## Exercise 244

### Determination of the coefficient of self-induction of a coil and the capacity of a condenser

#### 1. Determination of ohmic resistance of a coil

Measurement Number, $i$		1	2	3
Voltage, $U_i$	[V]			
Electric current, $I_i$	[mA]			
Resistance, $R_i$	[ $\Omega$ ]			

#### 2. Determination of the impedance and the coefficient of self-induction of a coil

Measurement Number, $i$		1	2	3
Voltage, $U_i$	[V]			
Electric current, $I_i$	[mA]			
Impedance, $Z_i$	[ $\Omega$ ]			
Coefficient of self-induction, $L_i$	[H]			
Average value of coefficient of self-induction, $L$ [H]				

#### 3. Determination of the impedance and the capacity of a capacitor

Measurement Number, $i$		1	2	3
Voltage, $U_i$	[V]			
Electric current, $I_i$	[mA]			
Impedance of capacitor, $Z_i$	[ $\Omega$ ]			
Capacity of capacitor, $C_i$	[ $\mu$ F]			
Average value of the capacity of capacitor		[ $\mu$ F]		

## Exercise 244. Determination of the coefficient of self-induction of a coil and the capacity of a condenser

### Introduction

#### A coil (winding) in a DC and AC circuit (direct and alternating current).

If we measure the intensity of the current flowing through the coil in a DC circuit (Fig. 1a) with voltage  $\mathcal{E}_a$  and in an alternating current circuit (Fig. 1b) with the same value of the RMS (root mean square voltage)  $\mathcal{E}_b$ , it will turn out that value  $I_b$  is clearly smaller than  $I_a$ . This means that the resistance of the coil in a DC circuit is less than in an AC circuit.

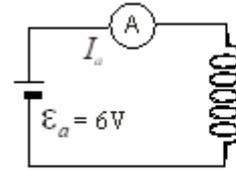


Fig. 1a

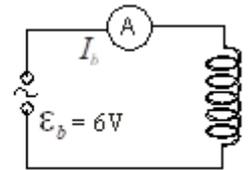


Fig.1b

A coil connected to a DC circuit behaves like an ohmic resistance whose value (resistance)  $R$  depends only on the geometrical dimensions and the type of conductor:

$$R = \rho l / S ,$$

$\rho$  — resistivity (also called specific electrical resistance or volume resistivity),  $l$  — is the length of the conductor,  $S$  — is the cross-sectional area of the conductor measured in square meters ( $m^2$ ). After connecting the coil to the AC circuit, additional *inductive resistance appears, related to the phenomenon of self-induction*.

When current flows through the coil, the magnetic field which strength is proportional to the current is generated inside the coil. The alternating current creates an alternating magnetic field which induces an electromotive force (EMF) of self-induction in the coil. EMF of self-induction, as a particular case of electromagnetic induction, is determined by the formula:

$$E_s = -L \frac{dI}{dt} ,$$

when  $dI/dt$  — the rate of change of current,  $L$  — the coefficient of self-induction. The coefficient self-induction is numerically equal to the (EMF) excited in the circuit where there is a change in current of 1 A in 1 s. The SI unit  $L$  is *Henry* marked as [H];  $1H = 1V \cdot s \cdot A^{-1}$ .

The  $L$  value depends on the geometric shape of the circuit, its size, and the magnetic permeability of the medium. For a long solenoid (cylindrical coil):

$$L = \frac{\mu_0 \mu_r N^2 S}{l} ;$$

$N$  — the number of turns,  $l$  — solenoid length,  $S$  — a cross sectional area of a solenoid,  $\mu_0$  — magnetic permeability of free space (a vacuum),  $\mu_r$  — relative magnetic permeability of the medium inside the solenoid.

According to Lenz's rule, the self-induction current at any time tries to counteract the change in the current flowing in the circuit, and therefore the coil exhibits additional resistance to self-induction  $R_L$ . In the case of an external source of sinusoidal alternating current with a circular frequency  $\omega$ , the inductive reactance (called the *inductance*) of a coil with a coefficient of self-induction  $L$  is given by the formula:

$$R_L = L \omega , \tag{1}$$

where  $\omega = 2 \pi f$ ,  $f$  — frequency of current changes ( $f = 50$  Hz).

If the ohmic resistance of an inductor is  $R$  then its total resistance  $Z$ , that called impedance or valence, in an AC circuit, we calculate as the geometric sum of the resistance  $R$  and the inductive resistance  $R_L$  :

$$Z = \sqrt{R^2 + R_L^2} = \sqrt{R^2 + (\omega L)^2} \quad (2)$$

### The capacitor in an AC circuit.

If we connect a capacitor in series with a light bulb to a DC source, Fig. 2, the light bulb will only flash because the current flows when the capacitor is charging. After the capacitor plates are charged, the current does not flow - in the DC circuit. The capacitor provides an almost infinitely large resistance. The

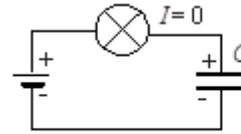


Fig.2

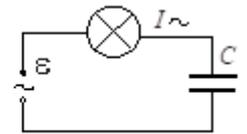


Fig.3

case is different for the AC source - Fig. 3. When the current in the circuit changes, the charge on the capacitor plates also changes. It can be positive or negative on a given plate, depending on the direction of the current flow. The alternating charging and discharging of a capacitor in a circuit allows the current to flow. The current amplitude is proportional to the capacitor capacity  $C$ , according to the definition of power. The greater the capacity, the greater the charge  $Q$  can be stored on the plates. Therefore, the capacitive resistance  $R_C$  is smaller, the greater the capacity  $C$ . For a sinusoidal alternating current source, it is expressed by the formula::

$$R_C = \frac{1}{\omega C}. \quad (3)$$

We can see that the capacitance is also inversely proportional to the angular frequency of the current -  $\omega$ . The higher the value  $\omega$ , the less charge will have time to accumulate on the capacitor during charging and the lower the voltage that counteracts the current flow.

If an AC circuit has an ohmic resistance  $R$  and a , we calculate the total resistance, or impedance, of the circuit using the formula:

$$Z = \sqrt{R^2 + R_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}. \quad (4)$$

### A coil and capacitor in an alternating current circuit.

In the general case, if the circuit consists of an ohmic resistance  $R$ , a coil with a self-induction coefficient  $L$  and a capacitor with a capacitance  $C$  (the elements are connected in series), then the impedance of such a circuit is determined by the formula:

$$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2}.$$

For AC circuits, Ohm's law is valid for the effective values of voltage  $U_s$  and current  $I_s$ . This means that the  $Z$  impedance satisfies the relation:

$$Z = \frac{U_s}{I_s}.$$

**Performance of the task**

**1. The determination of ohmic resistance of a coil**

Connect the circuit according to the diagram in figure 4.

Set the appropriate measurement range for the instruments (as instructed by your instructor). Turn on the DC power supply and take current and voltage readings; measure 3 times by changing the setting of the potentiometer that regulates the output voltage of the power supply. Calculate the ohmic resistance:

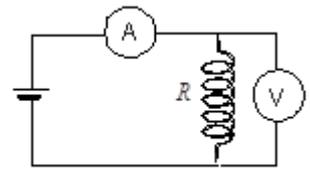


Fig.4

$$R_i = U_i / I_i, \quad i = 1, 2, 3. \quad (5)$$

**2. The determination of the coefficient of self-induction of a coil.**

Switch the power supply and universal meters to AC current measurement. Record the RMS (root mean square) voltage and current for the three output voltages of the power supply (numerically similar to the voltages selected for direct current). Calculate the impedance:

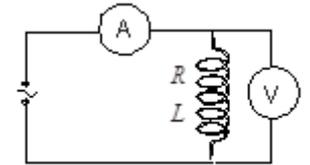


Fig.5

$$Z_i = U_i / I_i, \quad i = 1, 2, 3. \quad (6)$$

Transforming the relation (2) we obtain the formula for the coefficient of self-induction:

$$L_i = \frac{1}{\omega} \sqrt{Z_i^2 - R_i^2} \quad (7)$$

Based on the data from three measurements, calculate the average value of the L coefficient.

**3. The determination of the capacitance of the capacitor**

Connect the circuit as shown in Fig. 6. Determine the impedance of the circuit. Calculate the capacitance of a capacitor, substituting into equation (4) the value  $R=0$  (connection resistances are small)

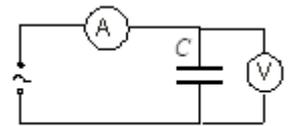


Fig.6

$$C_i = (\omega Z_i)^{-1}, \quad i = 1, 2, 3. \quad (8)$$

Calculate the average value of the capacitance C.

**Calculation of the uncertainties**

Maximum absolute error of a single measurement  $R, Z, L$  i  $C$  we can calculate based on the total differential method, which we will apply successively to the formulas (5), (6), (7) i (8). We treat the circular frequency  $\omega$  as a quantity not burdened by a measurement error.

In the case of indicating meters, the errors  $\Delta U_i, \Delta I_i$  are calculated from the instrument K class;

$$\text{np. } \Delta U = \frac{K \cdot Z_u}{100}, \quad Z_u \text{ — maximum voltage value for the given range}$$

For digital meters, the accuracy is equal to 1 % of the measured value for direct current and 1.5 % of the measured value for alternating current.

It is better to round up the result of the error calculation.

**1. Resistance  $R_i$  :**

$$\Delta R_i = R_i \left( \frac{\Delta U_i}{U_i} + \frac{\Delta I_i}{I_i} \right).$$

**2. Impedance  $Z_i$  :** 
$$\Delta Z_i = Z_i \left( \frac{\Delta U_i}{U_i} + \frac{\Delta I_i}{I_i} \right).$$

**3. Coefficient of self-induction (Inductance)  $L_i$  :** 
$$\Delta L_i = \frac{1}{L_i \omega^2} (Z_i \Delta Z_i + R_i \Delta R_i).$$

**4. Capacity of a capacitor  $C_i$  :** 
$$\Delta C_i = C_i \frac{\Delta Z_i}{Z_i}.$$

**Warning:** The value  $\Delta Z$  is not the same for a capacitor and for a coil. Determine errors for one of three measurements of a physical quantity.