Department of Physics and Biophysics; Institute of Biology

First name .... Date ....

Last name .... Degree program name ....

Exercise 367

Measuring Wavelength with a Diffraction Grating

Wavelength of the monochromatic light [nm]	
The distance between the grating and the screen l [m]	

I. Determination of the grating constant d

spectrum order	Distance from the slit [m]			Angle deflection	Grating constant	Average grating constant
	on the right	on the left	average			
n	x_p	x_l	x_n	α_n [°]	d_n [nm]	d [nm]

II. Determination of the wavelength of light

The colour of the spectral line		blue $\lambda_{tab} = 435{,}1$ nm			green $\lambda_{tab} = 546{,}1$ nm			yellow $\lambda_{tab} = 578{,}0$ nm		
Spectrum order, n		1	2	3	1	2	3	1	2	3
Distance from the slit [m]	on the right a_p									
	on the left, a_l									
	average, a_n									
Sine of the reflection angle α_n										
Wavelength, λ_n [nm]										
Average wavelength [nm]		$\lambda_I =$			$\lambda_{II} =$			$\lambda_{III} =$		
$\Delta\lambda_{tab} = \lvert\lambda - \lambda_{tab}\rvert$										
$(\Delta\lambda_{tab}/\lambda_{tab}) \cdot 100\%$										

Exercise 367. Measuring Wavelength with a Diffraction Grating

Introduction

Visible light is the narrow segment of electromagnetic spectrum (radiation in the form of waves of electric and magnetic energy) to which the human eye responds. Visible light wavelengths ranges (in vacuum) from $3{,}8 \cdot 10^{-7}$ m (beginning of violet light, frequency approx. $8{\cdot}10^{14}$ Hz) to $7{,}7 \cdot 10^{-7}$ m (end of red light, frequency approx. $4{\cdot}10^{14}$ Hz). Apart from visible spectrum, light also includes infrared and ultraviolet radiation.

Wavelength λ is equal to the distance between the adjacent parts of wave in the same phase (in the case of electromagnetic waves, this means that the vectors of the electric $\vec{E}$ (or magnetic $\vec{H}$) field have the same direction, value and sense, i.e. they are identical). The time T it takes a wave to travel the distance equal to the wavelength is called the *period* of the wave:

$$\lambda = c \cdot T = c/f \ ,$$

where c —speed of light (in vacuum 300 000 km/s), f — wave frequency (the number determined by the number of wavelengths passing along the path of the wave per unit of time).

The perception of light in the eye is related to the change in the electric field of the electromagnetic wave. The change in the value of the electric field intensity E in time, at a point distant by r from the light source, for a wave with frequency f can be expressed by the equation:

$$E = E_0 \sin\left[2\pi f\left(t - \frac{r}{c}\right) + \delta\right],$$

where E_0 is the amplitude, δ — the phase of the incipient wave

Light consists of a dual nature, meaning it has both particle and wave attributes. It is assumed that light consists of photons, which are particles with wavelike properties. The wave nature of light is displayed in such phenomena as *diffraction and interference of light rays*.

Diffraction and interference of light can be observed using a diffraction grating. The simplest diffraction grating is manufactured by curving a transparent glass plate with a sharp tool in a large number of evenly spaced parallel lines. The untouched gaps between the lines play role of slits. The distance between the slits is called the *diffraction grating constant d*. A diffraction grating is used for spectral analysis and measurement of the wavelength of light.

The light passing through the diffraction grating bends according to the Huygens's principle. Huygens's principle states that every point of wavefront on each slit may be regarded as a source of new wavelets that propagate in all directions. The phenomenon of bending of waves around the obstacles or the edges of the diaphragms (with dimensions comparable to the wavelength) is called *diffraction.* The light diffracted into semicular waves at the slits undergo *interference*. Wave interference is the superposition of waves of the same frequency, resulting in the amplitude of the new wave equal to the sum of the amplitudes of the individual waves. This can lead to the increase of the intensity of the resulting wave (constructive interference) or the decrease of the intensity (destructive interference), depending on how the "peaks" and "valleys" of the waves are related. Pure constructive interference occurs where the waves are in phase and pure destructive interference, where the waves are out of phase. If the light falls onto the screen, bright lines (constructive interference) and dark lines (destructive interference) can be observed.

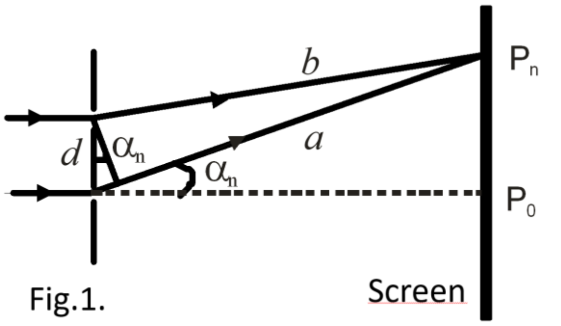

Fig.1.

If the paths taken by the two waves differ $a-b=d\sin\alpha_n$ by any integral number of wavelength, then constructive interference occurs (Fig. 1):

$$d\sin\alpha_n = n\lambda, \qquad (1)$$

where d – distance between slits (grating constant), α_n – deflection angle, n – order (number of the row), λ – wavelength of light

Equation (1) indicates that lines corresponding to different wavelengths will be seen at different places on the screen. By measuring the deflection angle α_n for the n line, we can determine the wavelength if we know the grating constant.

The wavelengths in opposite phases will extinguish mutually on the screen and we will get a dark line on the screen. In more detail, if the paths taken by the two waves differ by odd half-integral number of wavelengths, then the destructive interference occurs:

$$d\sin\alpha_n = (2n+1)\frac{\lambda}{2}.$$

A clear diffraction pattern (sharp light and dark lines on the screen) is obtained only when the grating constant is comparable to the wavelength of the diffracted light. In typical diffraction gratings, the number of scratches per 1 mm ranges from about 1200 (for ultraviolet light) to 300 (for infrared light).

Performance of the task

I. ***Equipment setup***

The scheme of the equipment setup is shown in Fig. 2. The light emitted by the S source passes through the slit in the screen and reaches the diffraction grating, placed on the stand at a distance of l from the screen. The diffraction grating is set parallel to the screen, and the scratches should be at the height of the slit (the direction of the scratches, like the slit in the screen, must be vertical). Behind the diffraction grating, the lens of the observer's eye creates an image of the diffracted picture on the retina. The observer will see a series of stripes of different colours on the right and left sides of the slit.

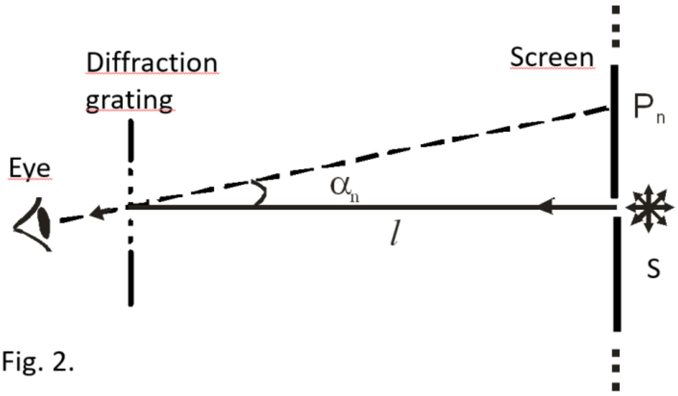

Fig. 2.

II. ***Determining the grating constant d***

1. Turn on a monochromatic light source (laser) with a known wavelength – λ_s.
2. Measure the distance from the central slit in the screen to the spectral lines for several orders on the left and right sides of the slit. Calculate the average distances of the line from the slit for each order:

$$x_n = (x_l + x_p)/2.$$

3. Calculate the sine of the deflection angle α, (see Fig. 2):

$$\sin\alpha_n = \frac{x_n}{\sqrt{x_n^{\ 2} + l^2}} \qquad (2)$$

4. Based on the formula (1), we can write

$$d_n = \frac{n\lambda_s}{\sin\alpha_n}. \qquad (3)$$

Substituting the values into formula (3) determine the grating constant for each row d_n.

5. Calculate the average value of the grating constant *d*:

$$d = \left(d_1 + d_2 + d_3\right)/3.$$

III. Determination of the wavelength of light λ

1. Place a mercury lamp behind the slit in the screen.
2. Measure the distance of the spectral lines: to the left of the slit -a_l and to the right -a_p. Make the measurements for three orders of intensely blue, green, and yellow lines.
3. For each order calculate the average distance of the line from the slit:

$$a_n = \left(a_l + a_p\right)/2.$$

4. Calculate the sine of the deflection angle:

$$\sin\alpha_n = \frac{a_n}{\sqrt{a_n^{\ 2} + l^2}}, \qquad (4)$$

and wavelength:

$$\lambda_n = \frac{d\sin\alpha_n}{n}. \qquad (5)$$

5. For each colour of line, calculate the average value of the wavelength: $\lambda = \left(\lambda_1 + \lambda_2 + \lambda_3\right)/3$.

Calculation of the uncertainties

The errors analysis concerns the measurement of the wavelength λ_n for the *n* order. The maximum uncertainty of measurement $\Delta\lambda_n$ is determined by differential calculus. According to the formula (5), the variables affected by the measurement uncertainty are the grating constant *d* and the deflection angle α_n :

$$\Delta\lambda_n = \left|\frac{\partial\lambda_n}{\partial d}\right|\Delta d + \left|\frac{\partial\lambda_n}{\partial\alpha_n}\right|\Delta\alpha_n \quad \Rightarrow \quad \Delta\lambda_n = \lambda\left(\frac{\Delta d}{d} + \frac{\Delta\alpha_n}{\mathrm{tg}\,\alpha_n}\right) = \lambda\left(\frac{\Delta d}{d} + \frac{l}{a_n}\Delta\alpha_n\right).$$

For Δd we accept the maximum error of the mean $\Delta d = \max\left|d - d_n\right|$; $n = 1, 2, 3$.

$\Delta\alpha_n$ we calculate using differential calculus. Because $\mathrm{tg}\,\alpha_n = \frac{a_n}{l}$, where $\alpha_n = \mathrm{arc\ tg}\frac{a_n}{l}$, we will get:

$$\Delta\alpha_n = \left|\frac{\partial\alpha_n}{\partial a_n}\right|\Delta a_n + \left|\frac{\partial\alpha_n}{\partial l}\right|\Delta l \quad \Rightarrow \quad \Delta\alpha_n = \frac{l\cdot a_n}{l^2 + a_n^{\ 2}}\left(\frac{\Delta a_n}{a_n} + \frac{\Delta l}{l}\right).$$

Substitute: $\Delta a_n = 2$ mm (measurement uncertainty of the grating distance), $\Delta l = 5$ mm.

Carry out the calculations for one order of the spectrum of one of the colour.